P2 Logarithms and Exponentials: exam questions

10 questions · 33 marks · full mark schemes

Name _____ Date _____ Score / 33

- a^x and $\log_a x$, laws of logarithms, solving $a^x = b$, change of base (natural logs and e^x are P3).
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p2/logs-exponentials-questions

1. **2 marks · warm-up**

Describe the single geometric transformation that maps the graph of $y = 4^x$ onto the graph of $y = 4^{x+2}$.

2. **2 marks · warm-up**

Solve the equation $4^x = 35$, giving your answer to 3 significant figures.

3. **2 marks · standard**

Simplify fully the expression $\dfrac{\log_a 81}{\log_a 3}$.

4. 3 marks · standard

A quantity Q grows exponentially: $Q = 200e^{0.05t}$. Find the value of t when $Q = 500$, to 3 sf.

5. 4 marks · standard

A curve has the equation $y = ab^x$, where a and b are constants and $b > 0$. The curve passes through the points $(1, 10)$ and $(3, 40)$. Find the values of a and b.

6. 4 marks · standard

The size of a population of bacteria, P, is modelled by the equation $P = 400(1.05)^t$, where t is the time in hours. Find the time taken for the population to exceed 1000, giving your answer to 1 decimal place.

7. 4 marks · harder

Solve the equation $3^{x+2} = 7^{x-1}$, giving your answer to 3 decimal places.

8. 4 marks · harder

Solve the equation $9^x - 12(3^x) + 27 = 0$.

9. 4 marks · harder

Solve the equation $\log_2(2x + 1) - \log_2(x - 3) = 3$.

10. 4 marks · harder

Find the exact root of the equation $2\log_5 x = \log_5(3x + 10)$.

Answers and worked solutions

Question 1 — 2 marks

1. M1 for identifying the transformation as a translation.
2. A1 for specifying it is a translation by the vector $\begin{pmatrix}-2\\0\end{pmatrix}$ (or 2 units to the left).

Question 2 — 2 marks

1. M1 for taking logarithms of both sides to get $x\log 4 = \log 35$ (or writing $x = \log_4 35$).
2. A1 for calculating $x = 2.56$.

Question 3 — 2 marks

1. M1 for expressing 81 as a power of 3 and applying the power law: $\log_a 81 = \log_a(3^4) = 4\log_a 3$.
2. A1 for cancelling $\log_a 3$ to leave exactly 4.

Question 4 — 3 marks

Answer: 18.3

$e^{0.05t} = 2.5$ **M1**

$0.05t = \ln 2.5$ **M1**

$t = 18.3$ **A1**

Answer: 18.3

Question 5 — 4 marks

1. M1 for setting up the equations $ab = 10$ and $ab^3 = 40$.
2. M1 for dividing the equations to eliminate a: $b^2 = 4$.
3. A1 for stating $b = 2$ (since $b > 0$).
4. A1 for finding $a = 5$.

Question 6 — 4 marks

1. M1 for setting up the inequality or equation: $400(1.05)^t > 1000$.
2. M1 for dividing by 400: $(1.05)^t > 2.5$.
3. M1 for taking logarithms and rearranging: $t > \frac{\log 2.5}{\log 1.05}$.
4. A1 for finding $t = 18.8$ hours.

Question 7 — 4 marks

1. M1 for taking logarithms of both sides: $(x+2)\log 3 = (x-1)\log 7$.
2. M1 for expanding the brackets: $x\log 3 + 2\log 3 = x\log 7 - \log 7$.
3. M1 for isolating x terms on one side: $x(\log 7 - \log 3) = 2\log 3 + \log 7$.
4. A1 for evaluating $x = \frac{2\log 3 + \log 7}{\log 7 - \log 3} = 4.890$ (3 d.p.).

Question 8 — 4 marks

1. M1 for recognising a hidden quadratic in 3^x and rewriting: $(3^x)^2 - 12(3^x) + 27 = 0$.
2. M1 for factorising the quadratic in 3^x: $(3^x - 9)(3^x - 3) = 0$.
3. M1 for solving the individual exponential equations: $3^x = 9 \implies x = 2$.
4. A1 for finding the second solution: $3^x = 3 \implies x = 1$.

Question 9 — 4 marks

1. M1 for applying the division law to combine logs: $\log_2\left(\frac{2x+1}{x-3}\right) = 3$.
2. M1 for converting out of log form: $\frac{2x+1}{x-3} = 2^3 = 8$.
3. M1 for multiplying by the denominator and rearranging: $2x + 1 = 8(x-3) \implies 2x + 1 = 8x - 24$.
4. A1 for solving to get $6x = 25 \implies x = \frac{25}{6}$.

Question 10 — 4 marks

1. M1 for using the power law to rewrite the LHS: $\log_5(x^2) = \log_5(3x + 10)$.
2. M1 for equating arguments to form a quadratic: $x^2 - 3x - 10 = 0$.
3. M1 for solving the quadratic: $(x-5)(x+2) = 0$.
4. A1 for stating $x = 5$ as the only valid solution (rejecting $x = -2$).

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