P2 Integration: exam questions

10 questions · 47 marks · full mark schemes

Name _____ Date _____ Score / 47

- Definite integrals; area under a curve and between a curve and a line; the trapezium rule.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p2/integration-questions

1. **4 marks · warm-up**

Find the exact area of the region completely bounded by the curve $y = x^2 + 1$, the x-axis, and the vertical lines $x = -1$ and $x = 2$.

2. **4 marks · warm-up**

Use the trapezium rule with 4 strips to estimate the value of $\int_0^2 \frac{1}{x+1}\,dx$. Give your answer to 3 decimal places.

3. 3 marks · standard

A student uses the trapezium rule with 5 strips of width 1 to estimate $\int_0^5 x^3\,dx$ and obtains 162.5. Find the exact value of $\int_0^5 x^3\,dx$. State whether the trapezium rule gives an overestimate or an underestimate, and justify why by considering the shape of the curve.

4. 4 marks · standard

The curve $y = x^2 - 2x$ sits entirely below the x-axis between $x = 0$ and $x = 2$. Find the exact area of the finite region bounded by the curve and the x-axis.

5. 4 marks · standard

The velocity of a particle, v ms^{-1}, is given by $v = 3t^2 - 6t + 8$. Find the total distance travelled by the particle in the first 3 seconds, which is given by the formula $s = \int_0^3 v\,dt$.

6. 5 marks · standard

Find the exact area of the finite region bounded by the curve $y = x^2 - 5x + 7$ and the horizontal line $y = 3$.

7. 5 marks · harder

Given that P is a constant and $\int_1^5 (2Px + 7)\,dx = 4P^2$, show that there are two possible values for P and find these values.

8. 6 marks · harder

Find the exact total area enclosed between the curve $y = x(x-1)(x-2)$ and the x-axis.

9. 6 marks · harder

The line $y = x - 1$ meets the curve $y = (x-1)(x-5)$ at the points A and C. Find the exact area of the finite region bounded by the line and the curve.

10. 6 marks · harder

The curves $y = 2x^2 - 6$ and $y = 10 - 2x^2$ enclose a finite region. Find the exact area of this region.

Answers and worked solutions

Question 1

4 marks

1. M1 for correctly setting up the integral $\int_{-1}^{2}(x^2+1)\,dx$.
2. M1 for integrating to $\left[\frac{1}{3}x^3+x\right]_{-1}^{2}$.
3. M1 for substituting the limits: $\left(\frac{8}{3}+2\right)-\left(-\frac{1}{3}-1\right)$.
4. A1 for the correct area: $\frac{14}{3}-\left(-\frac{4}{3}\right)=6$.

Question 2

4 marks

1. M1 for calculating the strip width $h=\frac{2-0}{4}=0.5$.
2. M1 for generating the correct y-values for $x=0, 0.5, 1, 1.5, 2$: $1, \frac{2}{3}, 0.5, 0.4, \frac{1}{3}$.
3. M1 for substituting these values into the trapezium formula: $\frac{0.5}{2}\times\left[1+\frac{1}{3}+2\left(\frac{2}{3}+0.5+0.4\right)\right]$.
4. A1 for the final estimated value 1.117.

Question 3

3 marks

1. M1 for finding the exact value using integration: $\left[\frac{1}{4}x^4\right]_0^5=156.25$.
2. M1 for stating the rule yields an overestimate as $162.5>156.25$.
3. A1 for correctly justifying that the curve $y=x^3$ is convex (curves upwards) for $x>0$, meaning the straight lines of the trapezia lie entirely above the curve.

Question 4

4 marks

1. M1 for setting up the integral $\int_0^2(x^2-2x)\,dx$.
2. M1 for integrating to $\left[\frac{1}{3}x^3-x^2\right]_0^2$.
3. M1 for substituting the limits to evaluate the integral: $\left(\frac{8}{3}-4\right)-0=-\frac{4}{3}$.
4. A1 for taking the absolute value to state the positive area as exactly $\frac{4}{3}$.

Question 5

4 marks

1. M1 for stating the integral $\int_0^3(3t^2-6t+8)\,dt$.
2. M1 for integrating to $\left[t^3-3t^2+8t\right]_0^3$.
3. M1 for substituting the limit of $t=3$: $27-27+24$.
4. A1 for calculating the correct final distance: $24\,m$.

Question 6

5 marks

1. M1 for setting $x^2-5x+7=3 \implies x^2-5x+4=0$.
2. M1 for finding the intersection limits $x=1$ and $x=4$.
3. M1 for formulating the integral $\int_1^4(3-(x^2-5x+7))\,dx=\int_1^4(-x^2+5x-4)\,dx$.
4. M1 for integrating $\left[-\frac{1}{3}x^3+\frac{5}{2}x^2-4x\right]_1^4$.
5. A1 for finding the area by evaluating limits to exactly 4.5 (or $\frac{9}{2}$).

Question 7

5 marks

1. M1 for integrating to $\left[Px^2+7x\right]_1^5$.
2. M1 for substituting limits to get $(25P+35)-(P+7)=24P+28$.
3. M1 for equating this to $4P^2$ and forming the quadratic $4P^2-24P-28=0$.
4. M1 for simplifying and factorising: $P^2-6P-7=0 \implies (P-7)(P+1)=0$.
5. A1 for stating both values: $P=7$ and $P=-1$.

Question 8

6 marks

1. M1 for finding the roots at $x=0,1,2$ and setting up two separate integrals: $\int_0^1(x^3-3x^2+2x)\,dx$ and $\int_1^2(x^3-3x^2+2x)\,dx$.
2. M1 for expanding the brackets to get the polynomial and integrating to $\left[\frac{1}{4}x^4-x^3+x^2\right]$.
3. M1 for calculating the first area: $\frac{1}{4}-1+1=\frac{1}{4}$.
4. M1 for calculating the second area: $\left(\frac{16}{4}-8+4\right)-\frac{1}{4}=-\frac{1}{4}$.
5. M1 for taking the absolute magnitude of the second area.
6. A1 for the total area: $\frac{1}{4}+\frac{1}{4}=0.5$ (or $\frac{1}{2}$).

Question 9

6 marks

1. M1 for expanding the curve equation $y=x^2-6x+5$ and equating to the line: $x-1=x^2-6x+5$.
2. M1 for rearranging to $x^2-7x+6=0$ and solving to find intersections at $x=1$ and $x=6$.
3. M1 for setting up the integral $\int_1^6((x-1)-(x^2-6x+5))\,dx=\int_1^6(-x^2+7x-6)\,dx$.
4. M1 for integrating to $\left[-\frac{1}{3}x^3+\frac{7}{2}x^2-6x\right]_1^6$.
5. M1 for evaluating the expression at both limits.
6. A1 for calculating the exact area: $18-\left(-\frac{17}{6}\right)=\frac{125}{6}$.

Question 10

6 marks

1. M1 for equating the curves: $2x^2-6=10-2x^2$.
2. M1 for solving $4x^2=16$ to find the intersection points $x=-2$ and $x=2$.
3. M1 for setting up the integral $\int_{-2}^{2}((10-2x^2)-(2x^2-6))\,dx=\int_{-2}^{2}(16-4x^2)\,dx$.
4. M1 for integrating to $\left[16x-\frac{4}{3}x^3\right]_{-2}^{2}$.
5. M1 for substituting the upper limit to get $32-\frac{32}{3}=\frac{64}{3}$.
6. A1 for the final area calculation: $\frac{64}{3}-\left(-\frac{64}{3}\right)=\frac{128}{3}$ (or $42\frac{2}{3}$).

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