P2 Further Trigonometry: exam questions

10 questions · 36 marks · full mark schemes

Name _____ Date _____ Score / 36

- tan θ = sin θ / cos θ and $\sin^2\theta + \cos^2\theta = 1$; solving trigonometric equations in a given interval (degrees or radians), including quadratics in sin/cos/tan.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p2/further-trig-questions

1. **2 marks · warm-up**

Given that $\sin\theta = \frac{5}{13}$ and that θ is an obtuse angle, find the exact value of $\cos\theta$.

2. **2 marks · warm-up**

Find the exact value of $\sin^2 30° + \tan^2 60°$.

3. **3 marks · standard**

Prove that $(\sin x + \cos x)^2 \equiv 1 + 2\sin x \cos x$.

4. 3 marks · standard

Solve the equation $\cos(x - 45°) = 0.5$ for $0° \leq x \leq 360°$.

5. 3 marks · standard

Simplify the expression $\cos^4\theta - \sin^4\theta$ completely.

6. 4 marks · standard

Given that $\sin\theta = \frac{3}{5}$ and $\frac{\pi}{2} < \theta < \pi$, find the exact values of $\cos\theta$ and $\tan\theta$.

7. 4 marks · harder

Prove the identity $\frac{\cos x}{1-\sin x} + \frac{1-\sin x}{\cos x} \equiv \frac{2}{\cos x}$.

8. 5 marks · harder

Given that $3\cos\theta = 8\tan\theta$, form a quadratic equation in $\sin\theta$ and find the possible exact values of $\sin\theta$.

9. 5 marks · harder

Solve the equation $\sin x \tan x = 3$ for $0° \leq x \leq 360°$. Give your answers to 1 decimal place.

10. 5 marks · harder

Solve the equation $2\sin x - \frac{1}{\sin x} = 1$ for $0° \leq x \leq 360°$.

Answers and worked solutions

Question 1

2 marks

1. M1 for using the identity $\sin^2\theta + \cos^2\theta = 1$ to find $\cos^2\theta = 1 - (\frac{5}{13})^2 = \frac{144}{169}$.
2. A1 for selecting the negative root because θ is obtuse (2nd quadrant), giving $\cos\theta = -\frac{12}{13}$.

Question 2

2 marks

1. M1 for substituting the exact values: $(0.5)^2 + (\sqrt{3})^2$.
2. A1 for evaluating the expression to $0.25 + 3 = 3.25$ (or $\frac{13}{4}$).

Question 3

3 marks

1. M1 for fully expanding the squared bracket to $\sin^2 x + 2\sin x\cos x + \cos^2 x$.
2. M1 for grouping $\sin^2 x + \cos^2 x$ and identifying it as equal to 1.
3. A1 for writing the final simplified expression $1 + 2\sin x\cos x$, completing the proof.

Question 4

3 marks

1. M1 for finding the primary solutions for $(x - 45°)$: $60°$ and $300°$.
2. M1 for adding $45°$ to both solutions to solve for x.
3. A1 for stating the final values $x = 105°$ and $x = 345°$.

Question 5

3 marks

1. M1 for recognising and factoring the difference of two squares: $(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$.
2. M1 for applying the identity $\cos^2\theta + \sin^2\theta = 1$ to the second bracket.
3. A1 for the fully simplified expression $\cos^2\theta - \sin^2\theta$.

Question 6

4 marks

Answer: $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$

Since $\frac{\pi}{2} < \theta < \pi$, the angle θ is in the second quadrant. **B1**

In Quadrant 2, sine is positive, but cosine and tangent are negative.

Using the Pythagorean identity:

$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$. **M1**

Therefore, $\cos\theta = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$. **A1**

Using $\tan\theta = \frac{\sin\theta}{\cos\theta}$: $\tan\theta = \frac{3/5}{-4/5} = -\frac{3}{4}$. **A1**

Question 7

4 marks

1. M1 for creating a common denominator: $\frac{\cos^2 x + (1-\sin x)^2}{\cos x(1-\sin x)}$.
2. M1 for expanding $(1 - \sin x)^2$ to get $1 - 2\sin x + \sin^2 x$ in the numerator.
3. M1 for using $\sin^2 x + \cos^2 x = 1$ to simplify the numerator to $2 - 2\sin x$.
4. A1 for factorising the numerator to $2(1 - \sin x)$, cancelling with the denominator, and concluding with $\frac{2}{\cos x}$.

Question 8

5 marks

1. M1 for substituting $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to obtain $3\cos\theta = \frac{8\sin\theta}{\cos\theta}$.
2. M1 for multiplying through by $\cos\theta$ to get $3\cos^2\theta = 8\sin\theta$.
3. M1 for replacing $\cos^2\theta$ with $1 - \sin^2\theta$ and forming the quadratic $3\sin^2\theta + 8\sin\theta - 3 = 0$.
4. M1 for factorising the quadratic to $(3\sin\theta - 1)(\sin\theta + 3) = 0$.
5. A1 for stating the only valid solution is $\sin\theta = \frac{1}{3}$ and explicitly rejecting $\sin\theta = -3$.

Question 9

5 marks

1. M1 for substituting $\tan x = \frac{\sin x}{\cos x}$ to get $\frac{\sin^2 x}{\cos x} = 3$, and rearranging to $\sin^2 x = 3\cos x$.
2. M1 for substituting $1 - \cos^2 x$ for $\sin^2 x$ to form the quadratic $\cos^2 x + 3\cos x - 1 = 0$.
3. M1 for using the quadratic formula to find the roots $\cos x = \frac{-3 \pm \sqrt{13}}{2}$.
4. M1 for evaluating the valid root $\cos x = \frac{-3+\sqrt{13}}{2} \approx 0.30277$ (and rejecting the negative root).
5. A1 for finding the two valid angles $x = 72.4°$ and $x = 287.6°$.

Question 10

5 marks

1. M1 for multiplying the entire equation by $\sin x$ to remove the fraction: $2\sin^2 x - 1 = \sin x$.
2. M1 for rearranging into a standard quadratic equation: $2\sin^2 x - \sin x - 1 = 0$.
3. M1 for factorising the quadratic to $(2\sin x + 1)(\sin x - 1) = 0$.
4. A1 for finding the solution to $\sin x = 1$, giving $x = 90°$.
5. A1 for finding the solutions to $\sin x = -0.5$, giving $x = 210°$ and $330°$.

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