P2 Differentiation: exam questions

10 questions · 36 marks · full mark schemes

Name _____ Date _____ Score / 36

- Increasing and decreasing functions; stationary points and their nature; optimisation problems.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p2/differentiation-questions

1. **3 marks · warm-up**

Find the range of values for which the curve $y = 5 + 4x - x^2$ is increasing.

2. **4 marks · warm-up**

Find the coordinates of the stationary point on the curve $y = x^2 - 10x + 15$ and determine its nature.

3. **3 marks · standard**

The graph of $y = f(x)$ is a positive quadratic curve (U-shaped) with a single minimum turning point at $(2, -5)$. Describe the shape and key features of the graph of its gradient function, $y = f'(x)$.

4. 3 marks · standard

A curve $y = f(x)$ is a cubic polynomial with a local maximum at $x = -1$ and a local minimum at $x = 3$. Describe the shape and key features of the graph of $y = f'(x)$.

5. 4 marks · standard

Find the x-coordinates of the stationary points on the curve with equation $y = x^3 - 3x^2 - 9x + 2$.

6. 4 marks · standard

Given that the total surface area of a cylinder is $S = 2\pi r^2 + \frac{256\pi}{r}$, use differentiation to find the value of r that minimises the surface area.

7. 3 marks · harder

A particle moves along a straight line such that its displacement s metres from a fixed point after t seconds is $s = t^3 - 9t^2 + 24t + 5$. Find the values of t when the particle is momentarily at rest.

8. 4 marks · harder

A curve has equation $y = x^3 - 3x^2 + 3x - 5$. Show that the curve has exactly one stationary point and determine its nature.

9. 4 marks · harder

Find the exact coordinates of the turning point on the curve $y = x + \frac{4}{x}$ for $x > 0$, and determine its nature.

10. 4 marks · harder

Find the coordinates of the stationary points on the curve $y = x^4 - 2x^2$ and classify them.

Answers and worked solutions

Question 1

3 marks

1. M1 for calculating $\frac{dy}{dx} = 4 - 2x$.
2. M1 for applying the condition for an increasing function: $4 - 2x > 0$.
3. A1 for finding the solution $x < 2$.

Question 2

4 marks

1. M1 for finding $\frac{dy}{dx} = 2x - 10$ and setting it equal to 0 to get $x = 5$.
2. A1 for finding the corresponding y-coordinate: $y = 5^2 - 10(5) + 15 = -10$, so the point is $(5, -10)$.
3. M1 for finding the second derivative $\frac{d^2y}{dx^2} = 2$.
4. A1 for stating that since $\frac{d^2y}{dx^2} > 0$, the point is a local minimum.

Question 3

3 marks

1. M1 for stating that the derivative of a quadratic is a linear function (a straight line).
2. A1 for stating that the line intersects the x-axis at $x = 2$, corresponding to the stationary point.
3. A1 for concluding that the straight line must have a positive gradient, since the original curve's gradient goes from negative to positive.

Question 4

3 marks

1. M1 for stating that the derivative of a cubic is a quadratic curve (a parabola).
2. A1 for stating that the parabola intersects the x-axis at the roots $x = -1$ and $x = 3$.
3. A1 for observing that the gradient goes from positive to negative to positive, meaning the parabola is U-shaped (has a positive x^2 coefficient).

Question 5

4 marks

1. M1 for finding $\frac{dy}{dx} = 3x^2 - 6x - 9$.
2. M1 for setting the gradient to zero: $3x^2 - 6x - 9 = 0$.
3. M1 for dividing by 3 and factorising or solving the quadratic: $x^2 - 2x - 3 = 0 \implies (x - 3)(x + 1) = 0$.
4. A1 for the correct x-coordinates $x = 3$ and $x = -1$.

Question 6

4 marks

1. M1 for differentiating to find $\frac{dS}{dr} = 4\pi r - 256\pi r^{-2}$.
2. M1 for setting the derivative to zero: $4\pi r - \frac{256\pi}{r^2} = 0 \implies 4\pi r = \frac{256\pi}{r^2}$.
3. M1 for solving for r^3: $r^3 = \frac{256\pi}{4\pi} = 64$.
4. A1 for taking the cube root to find $r = 4$ cm.

Question 7

3 marks

1. M1 for finding the velocity expression $v = \frac{ds}{dt} = 3t^2 - 18t + 24$.
2. M1 for setting $v = 0$ (at rest) and factorising: $3(t^2 - 6t + 8) = 0 \implies 3(t - 2)(t - 4) = 0$.
3. A1 for finding the times $t = 2$ and $t = 4$ seconds.

Question 8

4 marks

1. M1 for finding $\frac{dy}{dx} = 3x^2 - 6x + 3$.
2. M1 for setting it to 0 and factorising: $3(x^2 - 2x + 1) = 0 \implies 3(x - 1)^2 = 0$. This confirms only one root exists, $x = 1$.
3. M1 for checking the second derivative $\frac{d^2y}{dx^2} = 6x - 6$, which equals 0 at $x = 1$, requiring gradient testing.
4. A1 for testing points around $x = 1$ (e.g., $x = 0 \implies \frac{dy}{dx} = 3$, $x = 2 \implies \frac{dy}{dx} = 3$). Since the gradient is positive on both sides, it is a point of inflection.

Question 9

4 marks

1. M1 for differentiating $\frac{dy}{dx} = 1 - 4x^{-2}$.
2. M1 for setting to 0: $1 = \frac{4}{x^2} \implies x^2 = 4 \implies x = 2$ (ignoring -2 since $x > 0$).
3. A1 for the coordinates $(2, 4)$.
4. A1 for calculating $\frac{d^2y}{dx^2} = 8x^{-3}$, and since $8(2)^{-3} = 1 > 0$, confirming it is a local minimum.

Question 10

4 marks

1. M1 for calculating $\frac{dy}{dx} = 4x^3 - 4x$ and setting to zero to factorise: $4x(x^2 - 1) = 0$.
2. A1 for identifying the three roots $x = 0, 1, -1$.
3. M1 for calculating the respective y-coordinates $(0, 0), (1, -1), (-1, -1)$ and checking the second derivative $\frac{d^2y}{dx^2} = 12x^2 - 4$.
4. A1 for fully classifying: $x = 0 \implies \frac{d^2y}{dx^2} = -4 < 0$ (local maximum) and $x = \pm 1 \implies \frac{d^2y}{dx^2} = 8 > 0$ (local minima).

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