P2 The Binomial Expansion: exam questions

10 questions · 32 marks · full mark schemes

Name _____ Date _____ Score / 32

- Binomial expansion of $(a + bx)^n$ for positive integer n; ${}^nCr$ notation; using expansions for approximations.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p2/binomial-expansion-questions

1. **2 marks · warm-up**

Expand $(4 + x)^4$ completely in ascending powers of x.

2. **2 marks · warm-up**

Calculate the exact value of $\frac{100!}{98!2!}$.

3. **3 marks · standard**

Find the coefficient of x^3 in the binomial expansion of $(1 - 4x)^6$.

4. **3 marks · standard**

In the expansion of $(1+kx)^7$, the coefficient of x^2 is 84. Find the possible values of the constant k.

5. **3 marks · standard**

A biased coin has a probability p of landing on heads. In 5 tosses, the probability of getting exactly 2 heads is given by $10p^2(1-p)^3$. Expand this expression fully into a polynomial in terms of p.

6. **3 marks · standard**

Find the coefficient of x^5y^2 in the binomial expansion of $(x-2y)^7$.

7. **4 marks · harder**

Show that the expansion of $(1+x)^4-(1-x)^4$ simplifies to $8x+8x^3$.

8. 4 marks · harder

Find the coefficient of x^4 in the binomial expansion of $(2x - \frac{1}{x})^8$.

9. 4 marks · harder

In the expansion of $(1 + ax)^4(1 - x)^3$, the coefficient of x is 5. Find the value of a.

10. 4 marks · harder

The binomial expansion of $(2 + kx)^n$, where n is a positive integer, begins $32 - 240x + \ldots$. Find the values of n and k.

Answers and worked solutions

Question 1 — 2 marks

1. M1 for writing out the terms: $4^4 + \binom{4}{1}4^3x + \binom{4}{2}4^2x^2 + \binom{4}{3}4^1x^3 + x^4$.
2. A1 for the simplified expansion: $256 + 256x + 96x^2 + 16x^3 + x^4$.

Question 2 — 2 marks

1. M1 for simplifying the factorials to $\frac{100\times99}{2\times1}$.
2. A1 for evaluating the expression to 4950.

Question 3 — 3 marks

1. M1 for identifying the correct term using the binomial formula: $\binom{6}{3}(1)^3(-4x)^3$.
2. M1 for evaluating the combination and the power: $20 \times (-64x^3)$.
3. A1 for the correct coefficient: -1280.

Question 4 — 3 marks

1. M1 for setting up the equation for the coefficient of x^2: $\binom{7}{2}k^2 = 84$.
2. M1 for simplifying to $21k^2 = 84 \implies k^2 = 4$.
3. A1 for stating both possible values: $k = 2$ and $k = -2$.

Question 5 — 3 marks

1. M1 for correctly expanding $(1-p)^3$ using binomial expansion: $1 - 3p + 3p^2 - p^3$.
2. M1 for multiplying this expansion by $10p^2$.
3. A1 for the fully simplified polynomial: $10p^2 - 30p^3 + 30p^4 - 10p^5$.

Question 6 — 3 marks

1. M1 for writing out the specific term: $\binom{7}{2}(x)^5(-2y)^2$.
2. M1 for expanding the combination and powers: $21 \times x^5 \times 4y^2$.
3. A1 for the correct coefficient: 84.

Question 7 — 4 marks

1. M1 for expanding $(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$.
2. M1 for expanding $(1-x)^4 = 1 - 4x + 6x^2 - 4x^3 + x^4$.
3. M1 for substituting these back into the expression: $(1 + 4x + 6x^2 + 4x^3 + x^4) - (1 - 4x + 6x^2 - 4x^3 + x^4)$.
4. A1 for correctly cancelling the even powers and adding the odd powers to yield $8x + 8x^3$.

Question 8 — 4 marks

1. M1 for writing the general term of the expansion: $\binom{8}{r}(2x)^{8-r}(-\frac{1}{x})^r$.
2. M1 for determining the required value of r by considering powers of x: $x^{8-r} \times x^{-r} = x^{8-2r}$. Setting $8 - 2r = 4 \implies r = 2$.
3. M1 for substituting $r = 2$ into the coefficients: $\binom{8}{2}(2)^6(-1)^2$.
4. A1 for the correct evaluation: $28 \times 64 \times 1 = 1792$.

Question 9 — 4 marks

1. M1 for finding the first two terms of $(1+ax)^4$: $1 + 4ax$.
2. M1 for finding the first two terms of $(1-x)^3$: $1 - 3x$.
3. M1 for multiplying to find the combined x coefficient: $(1)(-3x) + (4ax)(1) = (4a - 3)x$.
4. A1 for setting this equal to 5 and solving: $4a - 3 = 5 \implies 4a = 8 \implies a = 2$.

Question 10 — 4 marks

1. M1 for equating the first term to find n: $2^n = 32 \implies n = 5$.
2. M1 for formulating the expression for the second term: $\binom{5}{1}2^4(kx) = -240x$.
3. M1 for solving the equation for k: $5 \times 16 \times k = -240 \implies 80k = -240$.
4. A1 for finding $k = -3$.

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