P2 Algebraic Methods: exam questions

10 questions · 32 marks · full mark schemes

Name _____ Date _____ Score / 32

- Algebraic long division; factor and remainder theorems; proof by deduction, exhaustion and counter-example.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p2/algebraic-methods-questions

1. **2 marks · warm-up**

Find the remainder when $f(x) = x^3 - 2x^2 + 5x - 7$ is divided by $(x - 2)$.

2. **2 marks · warm-up**

Find the remainder when $2x^3 + x^2 - 4x + 1$ is divided by $(x + 1)$.

3. **3 marks · standard**

Simplify fully $\frac{6x^2 - 7x + 2}{4x^2 - 4x + 1}$.

4. 3 marks · standard

Find the quotient when $2x^3 - 3x^2 - 11x + 6$ is divided by $x + 2$.

5. 3 marks · standard

Prove that the sum of any three consecutive integers is always a multiple of 3.

6. 4 marks · standard

Find the quotient and the remainder when $3x^3 + 8x^2 - 4x + 5$ is divided by $x + 3$.

7. 3 marks · harder

Prove that for any positive real numbers a and b, $\frac{a}{b} + \frac{b}{a} \geq 2$.

8. 4 marks · harder

Find the quotient and remainder when $4x^3 - 6x^2 + 4x - 5$ is divided by $2x - 1$.

9. 4 marks · harder

Find all the roots of the equation $x^3 - 7x^2 + 14x - 8 = 0$.

10. 4 marks · harder

Let $f(x) = 3x^3 - 17x^2 + 18x + 8$. Use the factor theorem to fully factorise $f(x)$.

Answers and worked solutions

Question 1

2 marks

1. M1 for using the Remainder Theorem and substituting $x = 2$ into the expression: $f(2) = 2^3 - 2(2)^2 + 5(2) - 7$.
2. A1 for evaluating to get $8 - 8 + 10 - 7 = 3$.

Question 2

2 marks

1. M1 for identifying the need to evaluate $f(-1)$ and substituting: $2(-1) + 1 - 4(-1) + 1$.
2. A1 for calculating correctly: $-2 + 1 + 4 + 1 = 4$.

Question 3

3 marks

1. M1 for factorising the numerator to $(3x - 2)(2x - 1)$.
2. M1 for factorising the denominator to $(2x - 1)^2$.
3. A1 for cancelling the common factor $(2x - 1)$ to obtain $\frac{3x-2}{2x-1}$.

Question 4

3 marks

1. M1 for setting up the division and finding the first term $2x^2$.
2. M1 for subtracting to get the next step ($-7x^2 - 11x + 6$) and finding the next quotient term $-7x$.
3. A1 for the final quotient $2x^2 - 7x + 3$ with a remainder of 0.

Question 5

3 marks

1. M1 for defining three consecutive integers algebraically, e.g., $n, n + 1$, and $n + 2$, where n is an integer.
2. M1 for summing the expressions: $n + (n + 1) + (n + 2) = 3n + 3$.
3. A1 for factorising out 3 to show it is a multiple of 3: $3(n + 1)$, and stating a concluding sentence.

Question 6

4 marks

1. M1 for setting up long division and establishing the first term $3x^2$.
2. M1 for finding the next quotient term $-x$.
3. A1 for the exact quotient $3x^2 - x - 1$.
4. A1 for finding the correct remainder of 8.

Question 7

3 marks

1. M1 for starting from the known fact that a square must be non-negative: $(a - b)^2 \geq 0$.
2. M1 for expanding to $a^2 - 2ab + b^2 \geq 0$ and rearranging to $a^2 + b^2 \geq 2ab$.
3. A1 for dividing through by ab (stating $ab > 0$ since $a, b > 0$) to obtain $\frac{a}{b} + \frac{b}{a} \geq 2$.

Question 8

4 marks

1. M1 for finding the first term of the quotient, $2x^2$.
2. M1 for subtracting $4x^3 - 2x^2$ and determining the next quotient term $-2x$.
3. A1 for the final correct quotient $2x^2 - 2x + 1$.
4. A1 for identifying the remainder is -4.

Question 9

4 marks

1. M1 for using the factor theorem to find a root, testing values to find $f(1) = 1 - 7 + 14 - 8 = 0$, so $x = 1$ is a root.
2. M1 for dividing the cubic by $(x - 1)$ to find a quadratic quotient.
3. A1 for finding the correct quadratic quotient $x^2 - 6x + 8$.
4. A1 for factorising to $(x - 2)(x - 4)$ and stating all three roots $x = 1, x = 2, x = 4$.

Question 10

4 marks

1. M1 for identifying a factor using the factor theorem, e.g. evaluating $f(2) = 24 - 68 + 36 + 8 = 0$, hence $(x - 2)$ is a factor.
2. M1 for performing algebraic long division by $(x - 2)$.
3. A1 for the correct quadratic quotient $3x^2 - 11x - 4$.
4. A1 for factorising the quadratic completely to obtain $(x - 2)(3x + 1)(x - 4)$.

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