P1 Integration: exam questions

10 questions · 38 marks · full mark schemes

Name _____ Date _____ Score / 38

- Indefinite integration of x^n ($n \neq -1$) and sums of such terms; finding the constant of integration from a point on the curve.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p1/integration-questions

1. **2 marks · warm-up**

Find $\int (4x^3 + 2x - 3)\,dx$.

2. **2 marks · warm-up**

Find $\int 5x^{-3}\,dx$.

3. **3 marks · standard**

Find $\int \frac{2x^3 - 5}{x^2}\,dx$.

4. 3 marks · standard

Find $\int \frac{5}{2\sqrt{x^3}}\,dx$.

5. 4 marks · standard

Given that $\frac{dy}{dx} = 3x^2 + 4x$ and that $y = 10$ when $x = 2$, find an expression for y in terms of x.

6. 4 marks · standard

Find $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$.

7. 5 marks · harder

Given that $f'(x) = \frac{6x^3+1}{x^2}$ and $f(1) = 4$, find an expression for $f(x)$.

8. 5 marks · harder

A curve $y = f(x)$ has a gradient function given by $f'(x) = 6x + k$, where k is a constant. The curve passes through the points $(1, 6)$ and $(2, 15)$. Find the equation of the curve.

9. 5 marks · harder

The curve C with equation $y = f(x)$ has a minimum point at $(3, -4)$. The gradient function of C is given by $f'(x) = 2x + a$, where a is a constant. Find the value of a and the equation of the curve C.

10. 5 marks · harder

The curve C passes through the point $(3, 10)$. The gradient function of C is given by $\frac{dy}{dx} = \frac{(x^2+2)(x^2-3)}{x^2}$. Find the equation of the curve C.

Answers and worked solutions

Question 1 — 2 marks

1. M1 for correctly integrating at least two terms: $\frac{4x^4}{4} + \frac{2x^2}{2} - 3x$.
2. A1 for the fully correct and simplified expression $x^4 + x^2 - 3x + c$.

Question 2 — 2 marks

1. M1 for increasing the power to -2 and dividing: $\frac{5x^{-2}}{-2}$.
2. A1 for $-\frac{5}{2}x^{-2} + c$ (or $-\frac{5}{2x^2} + c$).

Question 3 — 3 marks

1. M1 for dividing to split the fraction: $2x - 5x^{-2}$.
2. M1 for integrating to get $\frac{2x^2}{2} - \frac{5x^{-1}}{-1}$.
3. A1 for the simplified expression $x^2 + \frac{5}{x} + c$ (or $x^2 + 5x^{-1} + c$).

Question 4 — 3 marks

1. M1 for expressing the integrand in index form: $\frac{5}{2}x^{-\frac{3}{2}}$.
2. M1 for integrating to get $\frac{5}{2}\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}$.
3. A1 for the simplified answer $-5x^{-\frac{1}{2}} + c$ (or $-\frac{5}{\sqrt{x}} + c$).

Question 5 — 4 marks

1. M1 for integrating $\frac{dy}{dx}$ to find $y = x^3 + 2x^2 + c$.
2. M1 for substituting $x = 2$ and $y = 10$ into their integrated expression: $10 = (2)^3 + 2(2)^2 + c$.
3. M1 for solving to find c: $10 = 8 + 8 + c \implies c = -6$.
4. A1 for the final equation $y = x^3 + 2x^2 - 6$.

Question 6 — 4 marks

1. M1 for rewriting the integrand with fractional indices: $x^{\frac{1}{2}} + x^{-\frac{1}{2}}$.
2. M1 for increasing powers by 1 to $\frac{3}{2}$ and $\frac{1}{2}$.
3. M1 for dividing by the new powers correctly: $\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}}$.
4. A1 for $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$.

Question 7 — 5 marks

1. M1 for splitting the fraction: $f'(x) = 6x + x^{-2}$.
2. M1 for integrating to find $f(x)$: $3x^2 + \frac{x^{-1}}{-1} + c$.
3. A1 for simplifying to $f(x) = 3x^2 - x^{-1} + c$.
4. M1 for substituting $(1, 4)$ to find c: $4 = 3(1)^2 - (1)^{-1} + c \implies 4 = 2 + c \implies c = 2$.
5. A1 for the final function $f(x) = 3x^2 - \frac{1}{x} + 2$.

Question 8 — 5 marks

1. M1 for integrating to find the general equation: $f(x) = 3x^2 + kx + c$.
2. M1 for substituting $(1, 6)$: $6 = 3 + k + c \implies k + c = 3$.
3. M1 for substituting $(2, 15)$: $15 = 12 + 2k + c \implies 2k + c = 3$.
4. M1 for solving the simultaneous equations to find k: subtracting gives $k = 0$, so $c = 3$.
5. A1 for the final equation $f(x) = 3x^2 + 3$.

Question 9 — 5 marks

1. M1 for using the property of a minimum point that $f'(3) = 0$.
2. A1 for solving $2(3) + a = 0 \implies a = -6$.
3. M1 for integrating $f'(x) = 2x - 6$ to get $f(x) = x^2 - 6x + c$.
4. M1 for substituting $(3, -4)$ into the curve equation: $-4 = 3^2 - 6(3) + c \implies -4 = 9 - 18 + c$.
5. A1 for finding $c = 5$ and stating the final equation $y = x^2 - 6x + 5$.

Question 10 — 5 marks

1. M1 for expanding the numerator to $x^4 - x^2 - 6$.
2. M1 for dividing by x^2 to obtain the separated terms $x^2 - 1 - 6x^{-2}$.
3. M1 for integrating to find the general equation $y = \frac{1}{3}x^3 - x + 6x^{-1} + c$.
4. M1 for substituting $(3, 10)$ to find c: $10 = \frac{1}{3}(27) - 3 + \frac{6}{3} + c \implies 10 = 9 - 3 + 2 + c \implies c = 2$.
5. A1 for the final equation $y = \frac{1}{3}x^3 - x + \frac{6}{x} + 2$.

More integration practice with instant feedback: alevelmathrevision.com/alevel-frontend/units/p1/integration-questions
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