EDEXCEL INTERNATIONAL A LEVEL · P1 PURE MATHEMATICS 1

P1 Differentiation: exam questions

10 questions · 32 marks · full mark schemes

Name _____ Date _____ Score / 32

- Gradient of a tangent as a limit; differentiating x^n and sums of such terms; gradients, tangents and normals; the second derivative.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p1/differentiation-questions

1. **2 marks · warm-up**

Find $f'(x)$ given that $f(x) = x^{-5}$.

2. **2 marks · warm-up**

Find the derivative of $y = 5x^2 - 3x + 7$ with respect to x.

3. **2 marks · standard**

Differentiate $y = 4\sqrt{x}$ with respect to x.

4. **2 marks · standard**

Find $\frac{dy}{dx}$ when $y = 2x^4 - 3x^3 + 5x - 1$.

5. 3 marks · standard

Find the coordinates of the turning point on the curve $y = 3x^2 - 12x + 5$ by using differentiation.

6. 3 marks · standard

Find $f'(x)$ given that $f(x) = x(x^2 - 5x + 2)$.

7. 4 marks · harder

Prove, from first principles, that the derivative of $f(x) = x^2$ is $2x$.

8. 4 marks · harder

Find the equation of the tangent to the curve $y = x^2$ that is parallel to the line $y = 4x - 1$.

9. 5 marks · harder

Find the equation of the normal to the curve $y = x^3 - 2x$ at the point where $x = 2$.

10. 5 marks · harder

The curve $y = ax^2 + bx + c$ passes through the point $(0, 4)$. The gradient of the curve at the point $(1, 3)$ is -2. Find the values of a, b, and c.

Answers and worked solutions

Question 1 — 2 marks

1. M1 for multiplying by the power -5 and subtracting 1 from the power.
2. A1 for $-5x^{-6}$.

Question 2 — 2 marks

1. M1 for differentiating the x^2 and x terms.
2. A1 for $10x - 3$.

Question 3 — 2 marks

1. M1 for rewriting as $y = 4x^{\frac{1}{2}}$ and multiplying by the power.
2. A1 for $2x^{-\frac{1}{2}}$ (or $\frac{2}{\sqrt{x}}$).

Question 4 — 2 marks

1. M1 for attempting to differentiate the polynomial, decreasing powers correctly on at least two terms.
2. A1 for the fully correct derivative $8x^3 - 9x^2 + 5$.

Question 5 — 3 marks

1. M1 for differentiating to get $\frac{dy}{dx} = 6x - 12$.
2. M1 for setting the gradient to 0 ($6x - 12 = 0$) and solving to find $x = 2$.
3. A1 for substituting $x = 2$ back into the original equation to find $y = -7$, giving coordinates $(2, -7)$.

Question 6 — 3 marks

1. M1 for expanding the brackets to get $x^3 - 5x^2 + 2x$.
2. M1 for differentiating the expanded expression.
3. A1 for $3x^2 - 10x + 2$.

Question 7 — 4 marks

1. M1 for stating the limit definition: $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$.
2. M1 for expanding the numerator to $x^2 + 2xh + h^2 - x^2$.
3. M1 for simplifying to $\lim_{h \to 0} \frac{2xh + h^2}{h}$ and factoring out h to get $\lim_{h \to 0}(2x + h)$.
4. A1 for concluding that as $h \to 0$, $2x + h \to 2x$, so $f'(x) = 2x$.

Question 8 — 4 marks

1. M1 for noting the gradient of the tangent must be 4 (since it is parallel to $y = 4x - 1$).
2. M1 for differentiating $\frac{dy}{dx} = 2x$ and setting it to 4: $2x = 4 \implies x = 2$.
3. M1 for finding the corresponding y-coordinate $y = 2^2 = 4$ and forming the line equation: $y - 4 = 4(x - 2)$.
4. A1 for simplifying to $y = 4x - 4$.

Question 9 — 5 marks

1. M1 for finding the y-coordinate at $x = 2$: $y = 2^3 - 2(2) = 4$. Point is $(2, 4)$.
2. M1 for differentiating to find $\frac{dy}{dx} = 3x^2 - 2$.
3. M1 for finding the tangent gradient $m_t = 3(4) - 2 = 10$, and hence the normal gradient $m_n = -\frac{1}{10}$.
4. M1 for setting up the normal equation: $y - 4 = -\frac{1}{10}(x - 2)$.
5. A1 for the correct equation, e.g. $10y + x = 42$ or $y = -0.1x + 4.2$.

Question 10 — 5 marks

1. M1 for substituting $(0, 4)$ into the equation to immediately find $c = 4$.
2. M1 for substituting $(1, 3)$ into the curve's equation (with $c = 4$) to get $a + b + 4 = 3 \implies a + b = -1$.
3. M1 for differentiating to get $\frac{dy}{dx} = 2ax + b$.
4. M1 for using the gradient condition at $x = 1$: $2a(1) + b = -2 \implies 2a + b = -2$.
5. A1 for solving the simultaneous equations ($2a + b = -2$ and $a + b = -1$) to find $a = -1$ and $b = 0$.

More differentiation practice with instant feedback: alevelmathrevision.com/alevel-frontend/units/p1/differentiation-questions

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